Between theory and practice: Teaching maths to elementary school students in Poland

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Resumo: O sucesso escolar das crianças em Matemática é um dos objetivos mais importantes da educação na Polónia. Os resultados da investigação relativos aos desempenhos escolares em Matemática não são otimistas. Podemos observar baixos níveis de compreensão das crianças sobre a utilidade da Matemática, da capacidade de utilização de estratégias pessoais na resolução de problemas e do uso prático de capacidades matemáticas. Este artigo apresenta a discussão deste problema num quadro psicológico e didático e analisa as causas ligadas à prática escolar do ensino da Matemática.

Palavras-chave: ensino de matemática, comunicação na sala de aula, erros de alunos, educação elementar.

Abstract: The development of children's school achievements in mathematics is one of the most important aims of education in Poland. The results of research concerning monitoring of school achievements in maths is not optimistic. We can observe low levels of children's understanding of the merits of maths, self-developed strategies in solving problems and practical usage of maths skills. This article frames the discussion of this problem in its psychological and didactic context and analyses the causes as they relate to school practice in teaching maths.

Keywords: teaching maths, classroom communication, pupils' mistakes, elementary education.

Résumé : La réussite scolaire des enfants en mathématiques est l'un des objectifs les plus importants de l'éducation en Pologne. Les résultats de la recherche concernant le contrôle des résultats scolaires en mathématiques n'est pas optimiste.

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Nous pouvons observer le faible niveau des enfants en ce qui concerne la compréhension de l'utilité des mathématiques, les stratégies d'auto-développement dans la résolution de problèmes et l'utilisation pratique des compétences en mathématiques. L'article porte sur la discussion de ce problème dans un cadre psychologique et didactique et sur l'analyse des causes liées à la pratique scolaire de l'enseignement des mathématiques.

Mots-clés: enseignement des mathématiques, communication en classe, erreurs des élèves, enseignement primaire.

1. THE IMPORTANCE OF MATHEMATICAL EDUCATION FOR CHILDREN'S DEVELOPMENT

Mathematics as an element of the school curriculum is often mistakenly treated as a collection of facts, formulas and algorithms, which require mostly rote learning. Meanwhile, in reality, mathematics is a type of human activity whose most characteristic feature is the intense involvement of intellectual processes in the solving of problems. In other words, mathematics is not about counting, but about thinking. The main goals of mathematical education include: mathematising, discovering, reasoning, communicating (Freudenthal, 1972). Following Freudenthal's words the main goal of this paper is to analyse to what extent teaching maths is in fact teaching counting or thinking. Polish elementary school practice is under consideration. My comparison is based on the results of nationwide research on the maths achievements of 9-years olds complimented by TIMSS research.

The crux of the analysis is the reasons for differences and similarities in research results. They were significant in typical and non – typical tasks solved by students and curriculum-related as well as non-curriculum related ones.

Referring to the above mentioned background, it is worth expanding on the psychological basis of maths teaching. It applies modern developmental psychology, especially constructivist and cognitive theories as well as interdisciplinary research results – neurocognitive ones and the analyses of the role of discourse and communication in child development.

According to J. Bruner (1986) there are two factors in the process of the recognition of the world:

- objective scientific explanation of the world order

- individual mind effort to understand and make sense of its subjective world in the context of other individual worlds

In his theory Bruner emphasised the significance of child intuition and non-academic notions. He warned against the premature introduction of formal knowledge and definitions to children. At the same time he promoted the idea of experience and exploration of the world

by children. J. Bruner has distinguished three modes of representation in cognitive development: enactive, iconic and symbolic ones which are accessible to a human being during their lifetime.

In early education in maths the shift from the enactive to the symbolic stage is far too fast. In Bruner's opinion, the education process should encourage the child's own activity to discover, autonomously recognise and structure his/her knowledge

The independent child's activity guarantees its better usage and understanding .

The teacher who engages children to engage in active dialogue and supports them in the cognitive process builds a kind of scaffolding. He/she creates the social context by which a pupil can learn to solve problems and cooperate with peers.

The role of social interaction in knowledge acquisition was also stressed by L. S.Wygotsky in his psychological theory. He emphasised the significance of the zone of proximal development which is prior to development and makes it a challenge instead of a series of well – known/familiar skills.

An interesting approach to the communication process which supports learning and development of thinking is presented by A. Sfard in her book *Thinking as communicating. Human development, the growth of discourses and mathematizing (*Sfard, 2008*)*. She is of the opinion that thinking is an individualised form of interpersonal communication. She introduces the term commognition and stresses that communication is a kind of activity in which the activity of A is continued by B. The meaning of communication in this discursive approach is not confined to the relation between sender and receiver. It is a complicated web of relations in which the activities of particular subjects constitute a part of the dynamic model.

In this approach communication is integrated into the community, but the communication process never determines reaction. The discourse becomes a kind of joint recognition, discovering notions, exploration and experience. Therefore, a pupil who is solving some mathematical problem may suggest many different strategies, all of which will be adequate to the communication situation, even if they are wrong from a mathematical point of view.

The significance of early cognitive experience and activity in discovering the world is also confirmed by modern neurobiological research. It shows that the brain cells of young children are twice as active as adult brain cells. This process last till 9-10 years of age.

If children do not face intellectual challenge and only follow the patterns imposed by teachers their brains do not develop in a traditional educational environment.

Through learning mathematics, the children can develop:

- critical and reflective thinking
- the perception of regularities and relationships
- problem-solving skills
- creation of models and synthetic formulas
- the formulation of hypotheses and their verification in practice
- the use of various strategies for a solution
- an explanation of the way in which a solution was achieved
- an assessment of the correctness of the solution
- arguing and generalizing
- the use of acquired skills and knowledge in a new situation.

Therefore, during the education process it is worth paying attention to the following issues:

- the creation of educational situations which enable children to use their cognitive ability and to construct their knowledge on their own
- the social nature of mathematics and offering opportunities for learning through the communication process
- boosting children's thinking during the solving of mathematical problems which should be constructed so as to pose an intellectual challenge for children and refer to their zone of proximal development
- encouraging children to search for their own strategy to solve the problem, to treat the solution proposed by the teacher or presented in the textbook as one of the possible solutions, but not the only right one
- discussion and verification by the pupils of various solution strategies, convincing each other about the correctness of their solutions
- intense activity during the solving of mathematical problems – using not only specific elements, but also less conventional ones such as drawings, pictograms, visualizations, situational models, diagrams illustrating relationships and dependencies, building blocks, various teaching aids
- creating opportunities to manipulate specific elements until the children understand the sense of the given mathematical operation and the problem-solving strategy, until they construct their own interpretations
- manipulation of mathematical symbols to achieve an understanding of the given mathematical concept, use of the teachers' notations and notations developed by the children
- solving problems in groups, negotiating solutions, looking for arguments and evidence that could convince a peer to accept the selected solution
- searching for individual solutions to mathematical problems, respecting the various approaches to the given task,
- the ability to perceive errors, to explain how they arose and to search for the strategy to arrive at the correct solution
- promote the interesting and original solutions proposed by the pupils, encourage them to develop their own mathematical problems or riddles to be solved
- use specific everyday situations as sources of problems and tasks that activate mathematical thinking

2. ANALYTICAL SKILLS OF PUPILS COMPLETING PRIMARY EDUCATION IN POLAND

The goals and tasks of mathematical education presented above, and the suggested methods for their effective achievement can be compared with the results of teachers' activity in the educational practice. I shall refer to the results of nationwide research into school achievements in the field of mathematics among pupils who complete elementary education. In Poland, elementary school lasts for 6 years and is divided into two phases: 3-year elementary education (integrated) and 3-year stage of primary school, with the curriculum

broken down by subject. Children commence mandatory education at the age of seven; as of 2014 this mandatory age will be lowered to six year.

The results of research presented in this text refer to achievements of children completing low primary education, which means they are 9 years old.

In the years 2006-2011 a research project was carried out in Poland, financed by the European Social Fund. It covered the monitoring of school achievement in children completing low primary education, regarding their language and mathematical skills.

The main goals of the research were as follows:

➢ Diagnosis of school achievements in language and mathematics
➢ Description of school achievement context: impact of school and family background
➢ Improvement in quality of education at primary school level

The research was conducted on a nationwide scale and covered four types of environments: countryside, small town, medium-sized town and large city. The numbers of participants in the subsequent years are presented below:

Quantitative research

2006 – 2500 pupils and their parents, 137 teachers and head- teachers

2008 – 4800 pupils, 262 teachers

2010 – 4800 pupils, 290 teachers

2011- 3800 pupils, 170 teachers

The quantitative research was supplemented by qualitative research in schools randomly selected from among schools where the pupils achieved above-average and below-average results in the skills assessment tests. In those schools, interviews were conducted with headmasters and teachers, and classes delivered for third grade pupils were monitored. In this way, the researchers attempted to identify factors which influenced the pupils' school achievements and which were tied to the teachers' work methods, their manner of communication with the pupils during the classes, the set educational goals and tasks as well their expectations regarding the pupils and their activity (Dąbrowski, 2009; 2011; Murawska & Żytko, 2012; Żytko & Dagiel, 2011).

Analysis of the research results can be used as the basis to determine the level of maths skills of third graders and the features that characterize the relevant educational practice.

What do we know about the maths skills of 9-year olds?

Calculation

➢ highly developed skill of calculating following the given pattern
➢ low level of understanding of the sense of maths calculations despite doing them
➢ low level of self-developed calculation strategies
➢ low level of practical usage of one's maths knowledge

Solving text problems

> ➢ highly developed skill of doing simple, typical tasks
> ➢ not well developed skill of doing non-typical tasks
> ➢ common false perception of solving text problems i.e. correct solution should cover all given numbers presented in the text
> ➢ common false perception of checking and verifying the result, automatically following the pattern

Logical reasoning and reading text containing numbers such as menu, price list, timetable

> ➢ highly developed skill of searching for information (diagrams, tables, charts etc.) and using it in practice (although not taught at school)
> ➢ surprisingly well-developed skill of recognizing and explaining rules and logic behind mathematical tasks

What do we know about maths teaching school practice?

Observations of classes in the schools participating in the research provided knowledge on the process of teacher-pupil communication, types of activities undertaken by the pupils, types of questions asked by teachers and children. The characteristic features of most maths classes include:

The lack of:

> ➢ teachers' initiatives supporting students motivation to learn mathematics
> ➢ opportunities for students to talk about maths
> ➢ intellectual challenges in the field of maths
> ➢ children's thinking during maths lessons

Aside from that, another visible issue are the teachers' preference to follow one scheme of problem solving and rare opportunities for children in school practice to look for individual problem-solving strategies. Table 1 presents the results of observations conducted during classes held in the third grade of primary school, regarding the types of activities undertaken by the children during classes of Polish language and maths. The observations were carried out in 2010 at 20 primary schools in Poland. The total time of observation was 3117 minutes during 70 courses – 33 of maths and 37 of Polish language. The total observation time of maths lessons was 1444 minutes.

The dominant form is the pupils' participation in the process of verification of their knowledge by the teacher, and the listening to teacher – that is, situations that do not require significant activity by the pupils. Situations in which the pupils are active, express their own opinions, discuss, undertake creative tasks are much rarer. More creative activity and discussion can be observed during language classes than during maths ones. The maths classes, according to the

observation results, involve primarily the children's participation in the checking of their knowledge by the teacher.

Table 1: Children's activity during maths lessons

Percentage of children's participation	Total – all schools 2010	Maths education 2010
Organisation arrangements	7.1	6.1
Checking knowledge	61.1	84.9
Listening to teacher	27.3	14.7
Discussion	3.7	0.1
Creative activity	2.3	0.4
Total time of observation (in minutes)	3117	1444

Table 2 presents the types of statements made by teachers during communication with pupils during maths lessons. The results reveal the frequency – in minutes – at which the various types of statements appeared during the observed classes. As shown above, the most frequent type of teachers' statement – made every 2.4 minutes – is the giving of instructions and checking the children's knowledge and understanding. The children's asking for explanation of the achieved solution or asking open questions occurs much less frequently - during maths classes such statements appear sporadically, every 160.4 minutes. The teachers very infrequently support the children in problem solving – more frequently, made impatient by the lack of answer, they prompt the right solution or answer the questions instead of the pupils.

Table 2: Teachers' statements during maths lessons

How often (in minutes) the teacher	Total – all schools 2010	Maths education 2010
Gives instructions	2.4	2.2
Asks questions checking the knowledge of children	3.4	2.9
Asks children to explain, to reason, etc. (open question)	43.9	160.4
Imposes answers, answers himself/herself	8.4	6.9
Supports children in their problem solving	97.4	72.2
Accepts children's suggestions	42.1	55.5
Total observation time (in minutes)	3117	1444

3. ANALYSIS OF PUPILS' ERRORS – EXAMPLES REPRESENTATIVES OF THE NON-TYPICAL PROBLEMS

The children's maths skills were studied in the research in following domains:

- calculation
- solving text problems
- logical reasoning and reading text containing numbers such as menu, price list, timetable

The exercises that the children had to perform during a maths test included typical ones, frequently practiced at school, and non-typical ones, which required the application of the acquired knowledge and skills in new situations, to solve a problem.

Below is one of the examples of non-typical problems:

There were 40 sparrows on the tree. Suddenly most of them flew away, apart from 8. How many sparrows stayed on the tree?

This problem was solved correctly by 30.8% third-grade pupils from the nationwide sample – which means the result is unsatisfactory. Almost 70% of the children were unable to deal with this problem. The problem is non-typical, because the correct answer is provided in the text –

it only needs to be found and captured. Some of the children succeeded and gave such answers as:

6. Na drzewie siedziało 40 wróbli. Nagle większość z nich, oprócz _8_, odleciała. Ile wróbli zostało na tym drzewie?
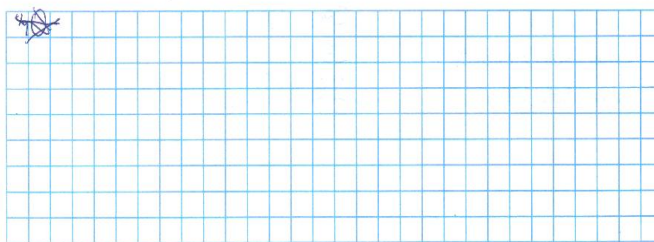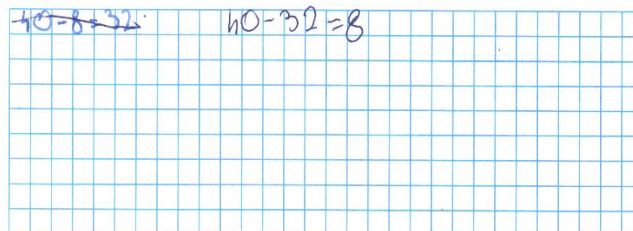
Odpowiedź: Na tym drzewie zostało 8 wróbli.

Figure 1: Answer *8 sparrows stayed on that tree*

In that case, the pupil underlined the number which is the correct answer in the text, and next gave a verbal answer. In another case, a child added their own comment, "calculation not necessary."

6. Na drzewie siedziało 40 wróbli. Nagle większość z nich, oprócz 8, odleciała. Ile wróbli zostało na tym drzewie?

40-8=32   40-32=8

Odpowiedź: Na drzewie zostało 8 wróbli

Figure 2: Answer *8 sparrows stayed on that tree* with a subtraction

In another example, a mathematical operation appeared – the child calculated mentally how many birds flew away and wrote down the subtraction whose result confirmed the answer. This child tried to prove that the provided result was not accidental.

However, almost half of the respondents – as much as 49.1% of the children solved the problem by performing the subtraction as presented in the example below.

Figure 3: Answer *32 sparrows stayed on that tree*

The pupils used the numbers provided in the text of the problem. It was stated that some of the sparrows flew away, so the subtraction has to be performed. This is one of the many examples of typical mistakes made by children, which proves that methodological stereotypes are present in the school practice and popularized by teachers during the process of mathematical education. The children are taught to solve the text problems not by looking for a solution for the actual problem, but by performing the right calculation which has to utilize all numbers provided in the text.

Aside from subtraction, division can also be performed, as shown in the example presented below:

Figure 4: Answer *5 sparrows stayed on that tree*

An interesting approach was found in the next example. The child came to the conclusion that the authors of the test made an error in the text of the problem and formulated the question in a wrong manner, so they needed to be corrected. The child crossed out the question given in the test and entered his/her own proposal, "How many sparrows flew away?"
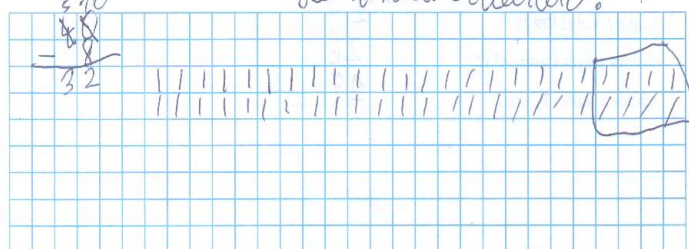
Figure 5: Answer *32 sparrows flew away*

It is also worth noting that in this example the child attempted a calculation – subtracting the smaller number from the bigger one, but did not manage to do it properly, so he/she drew a simple diagram which was the result of subtracting 8 from 40.

In order to verify the degree to which our research hypothesis, stating that sources of that error are linked to the manner of teaching, would be confirmed, in a subsequent run of the research a closed question was applied.

There were 30 sparrows on the tree. Suddenly most of them flew away, apart from 6. How many sparrows stayed on the tree?

The children were given 4 answers to choose from:

    A.   5
    B.   24
    C.   6
    D. 36

The results achieved were similar to the previous problem, asked in the open form: A – 4% of answers, B-41.9%, C – 50.2%, D – 3.3%. Half of the children provided the correct answer, but almost 42% made the same mistake as previously – they performed subtraction with the use of numbers provided in the text.

Analysis of the solutions shows clearly that changes are required in the process of teaching the children how to solve text-based tasks, because the results achieved so far do not show that the manner of teaching mathematics supports the development of the children's thinking. Quite the contrary, the thinking schemes are preserved by practicing typical tasks and exercises. This hypothesis is additionally strengthened by the analysis of statements made by teachers during the solving of text-based tasks in class. Here is one example.

The teacher formulates instructions such as:

Open your books on page 24, task 7.

Karol, will you read aloud for us?

What kind of data do we have in this task?

What kind of calculations do we have to apply?

…

Now we will do task number 9. This time we will have to multiply and divide.

This means that his statements stress the search for the appropriate calculation by the children – and not for the strategy to solve the problem contained in the task.

The second example of a text task which I shall present here will enable better understanding of the problem – also by describing the nature of mistakes made by the pupils.

The children had to solve the following task:

By the road where Camil lives trees were planted – one every 10 metres. The first one at the beginning, the last one at the end of the road. The road is 130 metres long. How many trees were planted?

Figure 6: Answer *13 trees were planted*

Most of the respondents performed the wrong calculations, using the numbers provided in the task. The children either divided or multiplied the numbers, as in the examples below:

9. Wzdłuż drogi, przy której mieszka Kamil, posadzono co 10 metrów drzewka.
   Pierwsze drzewko posadzono na początku drogi, a ostatnie na jej końcu.
   Droga ma 130 metrów długości. Ile drzewek posadzono?

$$130 \cdot 10 = 1300$$

Odpowiedź: _Pozostało 1300 drzewek do posadzenia._

Figure 7: Answer *There are 1,300 trees to be planted*

Only a few percent of the children attempted to use a drawing to solve this problem, in order to imagine the situation described in the task. This led them to the correct solution.

Below are two examples of tasks whose contents are similar to the previous ones, but solved correctly with the use of drawings made by the children.

6. Wzdłuż drogi, przy której mieszka Kamil, posadzono 13 młodych drzewek.
   Drzewka sadzono co 10 metrów. Pierwsze drzewko posadzono na początku drogi,
   a ostatnie na jej końcu. Jaką długość ma ta droga?

$$10 \cdot 12 \, m = 120 \, m$$
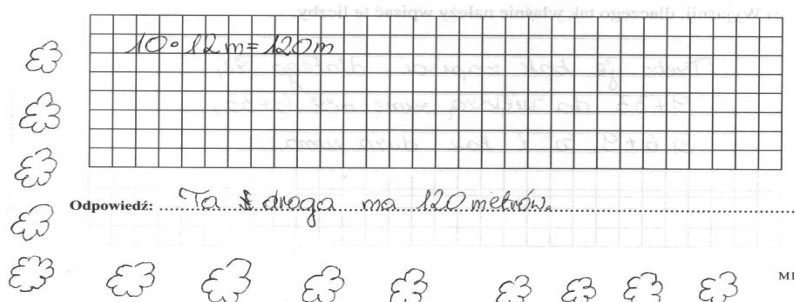
Odpowiedź: _Ta droga ma 120 metrów._

Figure 8: Resolution involving a calculation

6. Along the road where Camil lives, 13 young trees were planted. The trees were planted one every 10 metres, the first one at the beginning of the road and the last one at the end. What is the length of the road?

Figure 9: Resolution involving a drawing

9. Along the road where Camil lives, trees were planted, one every 10 metres. The first tree was planted at the beginning of the road, the last one at the end. The road is 150 metres long. How many trees were planted?

In both cases the children attempted to solve the problem contained in the task. In the second example the solution does not involve a calculation, but uses a drawing which shows the child's way of thinking.

SUMMARY

Analysis of the selected results of research conducted on Polish 9-year olds regarding their mathematical skills suggest the presence of a very disturbing phenomenon, of schematization of the children's thinking already at the early stages of school education. The teachers rarely offer the children the opportunity to develop their own strategies for problem solving – they prefer to provide the instruction for arriving at the solution, and next to practice that skill on a series of typical tasks. This does not support the development of the children's independent thinking, rather locks them within existing schemes. The educational practice is therefore closer to behavioural education models than to cognitive constructivism or the cognitive approach. This is certainly a reason for in-depth reflection on the ways of developing the children's mathematical skills, but also on the level of teachers' training and on the relationship between pedagogical theory and actual school practice. In order to increase teaching quality at the early school stages, analysis of the research results is required, and actions need to be undertaken that would change the school practice in that respect. The paradox of the current educational situation lies in the fact that the children have often better mastered those skills which the school did not work on – and therefore did not deprive them of the creative element in their thinking. This is confirmed by the international TIMMS (Trends in International Mathematics and Science Study, 2012) (Mullis et al., 2012) study of the mathematical, research and sciences skills, in which Poland participated for the first time in 2011. The Polish third graders came out with good results, above average in terms of data presentation (tables, charts, pictograms) – however, they did not acquire those skills at school, similarly as in the case of solving problem-based tasks. The national curriculum does not cover data presentation (charts, tables) and problem based tasks. The emphasis is put on calculation and practice in solving typical tasks following the given pattern. However, one

should be taken into account - the fact that the school nowadays is only one source of education. Children learn a lot outside school, especially via media, from different sources. The results achieved by Polish pupils in the area of arithmetic (numbers and calculation) are below average – and this is the type of skill that is practiced especially intensively at school. The total result of Poland in TIMSS is not very optimistic; we achieved 34th place in maths, last in Europe, among 50 countries which participated in this research.

The discrepancy is clearly visible but what is more, if we focus only on the area of the national curriculum covered the result for Poland is still the same- we come even lower in 35th place. It seems to prove the conclusion that maths skills practice at school is not sufficiently well-mastered to achieve better than average results.

The promotion of good educational practices is one of the ways of influencing educational change. The creation of and support for groups of teachers who work with children using the assumptions of constructivist theories should give positive results over time.

References

Bruner, J. (1986). *Actual minds, possible worlds.* MA: Harvard University Press.

Dąbrowski, M. (2009). *Research on the basic skills of a 9-year old in language and mathematics. A third grade pupil and his/her teacher – report from research 2008,* CKE Warsaw.

Dąbrowski, M. (2011). *The research on basic skills of 9-year old in language and mathematics. Third grade pupil 2010,* CKE Warsaw 2011.

Freudenthal, H. (1972). *Mathematics as an Educational Task. New York:* Springer.

Mullis, I. V. S., Martin, M. O., Minnich, C. A., Tanco, G. M., Arora, A., Centurino, V. A. S. & Castle, C. E. (Eds.) (2012). *TIMSS 2011 Encyclopedia. Education Policy and Curriculum in Mathematics and Sciences.* Boston: TIMSS & PIRLS Intenational Center, Lynch School of Education, Boston College.

Murawska, B. & Żytko, M. (2012). *The research on basic skills of 9-year old in language and mathematics. Pupil – his/her home and School.* CKE Warsaw.

Żytko, M. & Dagiel, M. (2011). *School life reality in the rural areas.* CKE Warsaw 2011.